RACCOLTA DI ESERCIZI PER I CORSI PRELIMINARI

IV PARTE: ESPONENZIALI E LOGARITMI

POTENZE AD ESPONENTE REALE

Tra la seguenti potenze, distinguere quelle che si possono calcolare da quelle che non hanno senso nel campo reale

1. 2^{π} **2.** $\pi^{\sqrt{2}}$ **3.** $\pi^{-\sqrt{2}}$ **4.** $(-\pi)^{\sqrt{2}}$

5. $(-\pi)^{-\sqrt{2}}$ **6.** $(2-\sqrt{3})^{2+\sqrt{3}}$ **7.** $(2+\sqrt{3})^{-2-\sqrt{3}}$ **8.** $(2-\sqrt{5})^{2+\sqrt{5}}$

9. $(\pi-3)^{2\sqrt{3}}$ **10.** $(6-2\pi)^{2\sqrt{\pi}}$ **11.** $(\sqrt{2}-1)^{2+\sqrt{7}}$ **12.** $\left(\frac{\sqrt{2}}{2}-1\right)^{\sqrt{7}}$

{*R.* non hanno senso nel campo reale le potenze 4, 5, 8, 10, 12}

Stabilire se le seguenti uguaglianze sono vere o false

13. $2^{2\sqrt{2}}=4$ **14.** $\left(\sqrt{2}\right)^{2\sqrt{2}}=2^{\sqrt{2}}$ **15.** $\left(5\sqrt{7}\right)^{\sqrt{2}}=5^{\sqrt{2}}\cdot\left(\sqrt{7}\right)^{\sqrt{2}}$

16. $\sqrt{3}^{\sqrt{3}^{\sqrt{3}}}=3\sqrt{3}$ **17.** $\left(\sqrt{3}^{\sqrt{3}}\right)^{\sqrt{3}}=3\sqrt{3}$ **18.** $2^{\pi+\sqrt{5}}=2^{\pi}\cdot 2^{\sqrt{5}}$

19. $(\pi+\sqrt{5})^{\sqrt{2}}=\pi^{\sqrt{2}}+\sqrt{5}^{\sqrt{2}}$ **20.** $\left(\sqrt{2}\right)^{2\sqrt{2}}=2^{\sqrt{2}}$ **21.** $\left(\sqrt{2}\right)^{4+\sqrt{2}}=4\cdot 2^{\sqrt{2}}$

{*R.* sono vere le uguaglianze 14, 15, 17, 18, 20, 21; sono false le uguaglianze 13, 16, 19}

LOGARITMI

In ciascuno dei seguenti esercizi sono assegnati un numero positivo x ed un numero b positivo e diverso da 1; calcolare in ciascun caso il logaritmo di x in base b

22. $x = 49$; $b = 7$ {*R.* 2} **23.** $x = 256$; $b = 4$ {*R.* 4}

24. $x = 256$; $b=\frac{1}{2}$ {*R.* –8} **25.** $x=\frac{1}{125}$; $b = 5$ {*R.* –3}

26. $x = 1000$; $b = 100$ {*R.* $\frac{3}{2}$} **27.** $x=\frac{1}{64}$; $b = 16$ {*R.* $-\frac{3}{2}$}

28. $x=\frac{1}{32}$; $b=\frac{1}{8}$ {*R.* $\frac{5}{3}$} **29.** $x = 27$; $b=\sqrt{3}$ {*R.* 6}

Aiutandosi anche con scomposizioni in fattori primi, semplificare le seguenti espressioni (per semplicità di scrittura, viene utilizzato il simbolo "Log" con il significato di "logaritmo decimale", cioè in base 10; in realtà, i risultati non cambiano se si sostituisce 10 con un'altra base)

30. Log 3 + Log 7 {*R.* Log 21} **31.** Log 10 – Log 12 {*R.* Log $\frac{5}{6}$}

32. $\text{Log } 24 + \text{Log } 10 - \text{Log } 36$ $\{R.\ \text{Log } \frac{20}{3}\}$

33. $\text{Log } \frac{7}{4} + \text{Log } \frac{2}{63} - \text{Log } \frac{32}{9}$ $\{R.\ -6 \text{ Log } 2\}$

34. $\text{Log } 32 + \text{Log } 33 + \text{Log } 34 - \text{Log } 35 - \text{Log } 36$ $\{R.\ \text{Log } \frac{1496}{105}\}$

35. $\text{Log } \frac{2}{27} - \text{Log } \frac{13}{10} + \text{Log } \frac{169}{45} - \text{Log } \frac{3}{13}$ $\{R.\ 2 \text{ Log } \frac{26}{27}\}$

36. $\frac{1}{2}\text{Log } 18 + \frac{3}{4}\text{Log } 32 - \frac{1}{4}\text{Log } 54 + \frac{1}{2}\text{Log } 45$ $\{R.\ 4 \text{ Log } 2 + \frac{5}{4}\text{Log } 3 + \frac{1}{2}\text{Log } 5\}$

37. $\frac{1}{3}\text{Log } \frac{2}{3} + \frac{1}{6}\text{Log } \frac{18}{5} + \frac{2}{3}\text{Log } \frac{25}{16}$ $\{R.\ \frac{7}{6}\text{Log } 5 - \frac{13}{6}\text{Log } 2\}$

Stabilire se le seguenti uguaglianze sono vere o false (tenere presente che le variabili indicano in generale numeri REALI, e che quando la base non è precisata va comunque intesa positiva e diversa da 1)

38. $\log_p p^2 = 2$

39. $\log_{\frac{1}{p}} p^3 = \frac{1}{3}$

40. $\log_{p+1}\left(p^3 + 3p^2 + 3p + 1\right) = 3$

41. $\log_{p^2-2p+1} \frac{1}{p^3 - 3p^2 + 3p - 1} = -\frac{3}{2}$

42. $\log_b\left(b^3 + \sqrt{b}\right) = \frac{7}{2}$

43. $\log_{\sqrt{b}}\left(b\sqrt{b\sqrt[5]{b^2}}\right) = \frac{17}{5}$

44. $\log_2\left(x^2 - 7x + 10\right) = \log_2(x-2) + \log_2(x-5)$

45. $\log_5 \frac{1}{\sqrt{5p + 3 - 2p^2}} = -\log_5(2p+1) - \log_5(3-p)$

$\{R.$ sono vere le uguaglianze 38, 40, 41, 43; sono false le uguaglianze 39, 42, 44, 45$\}$

In ciascuno dei seguenti casi, determinare i due numeri interi consecutivi tra i quali il logaritmo indicato è compreso

46. $\log_7 200$ $\{R.$ *tra 2 e 3*$\}$

47. $\log_{11} 1000$ $\{R.$ *tra 3 e 4*$\}$

48. $\log_\pi 9$ $\{R.$ *tra 1 e 2*$\}$

49. $\log_e 83$ $\{R.$ *tra 4 e 5*$\}$

50. $\log_{\sqrt{2}} \frac{3}{2}$ $\{R.$ *tra 1 e 2*$\}$

51. $\log_5 \frac{1}{50}$ $\{R.$ *tra −3 e −2*$\}$

52. $\log_{\frac{3}{7}} 5$ $\{R.$ *tra −2 e −1*$\}$

53. $\log_{\frac{4}{5}} \frac{9}{25}$ $\{R.$ *tra 4 e 5*$\}$

EQUAZIONI ESPONENZIALI

54. $3^x = 243$ $\{R.\ 5\}$

55. $2^x = 1$ $\{R.\ 0\}$

56. $7^x = \frac{1}{343}$ $\{R.\ -3\}$

57. $11^{2x+5} = \frac{1}{1331}$ $\{R.\ -4\}$

58. $\left(\frac{2}{3}\right)^{x-3} = \frac{243}{32}$ $\{R.\ -2\}$

59. $10^x = \frac{1}{100000}$ $\{R.\ -5\}$

60. $13^x = 87$ $\{R.\ \log_{13} 87\}$

61. $7^{x+2} = 44$ $\{R.\ \log_7 44 - 2\}$

62. $49^{3x+\frac{7}{2}}=101$ $\{R.\ \frac{\log_7 101-7}{6}\}$

63. $100^{5x-\frac{11}{3}}=81$ $\{R.\ \frac{11+6\log_{10}3}{15}\}$

64. $4^x=\frac{1}{\sqrt{2}}$ $\{R.\ -\frac{1}{4}\}$

65. $2^x\cdot 2^{\frac{1}{x}}=\sqrt[3]{1024}$ $\{R.\ 3\}$

66. $2^{x-1}\cdot 3^{x+1}=9$ $\{R.\ 1\}$

67. $2^{x-3}\cdot 4^{x-5}=\frac{1}{16}$ $\{R.\ 3\}$

68. $2^{\frac{1}{x}}\cdot 4^{\frac{1}{x+1}}=4$ $\{R.\ -\frac{1}{2}\ ;\ 1\}$

69. $2^{3x+\frac{1}{4}}\cdot 8^{\frac{x}{2}+\frac{1}{8}}=27$ $\{R.\ \frac{24\log_2 3-5}{28}\}$

70. $2^{5x-1}\cdot 9^{1+5\frac{x}{2}}=189$ $\{R.\ \frac{1+\log_6 7}{5}\}$

71. $125^{\frac{x+2}{3}}\cdot 3^x=\frac{5}{6}$ $\{R.\ -\log_{15}2-1\}$

72. $10^{x+3}-10^{2+x}+100^{\frac{2x+1}{2}}=910$ $\{R.\ 0\}$

73. $2^{4x-1}+4^{2x+\frac{3}{2}}-16^{x-1}=270$ $\{R.\ \frac{5}{4}\}$

74. $5^{x^2+x-2}+5^{x^2+x+1}=\frac{126}{25}$ $\{R.\ -1\ ;\ 0\}$

75. $2^{1-2\sqrt{x-3}}+4^{1-\sqrt{x-3}}=\frac{3}{2}$ $\{R.\ 4\}$

76. $2^{2x}-20\cdot 2^x+64=0$ $\{R.\ 2\ ;\ 4\}$

77. $6^{2x+2}+179\cdot 6^x-5=0$ $\{R.\ -2\}$

78. $7^{2x}-44\cdot 7^x-245=0$ $\{R.\ 2\}$

79. $9^x+189=10\cdot 3^{x+1}$ $\{R.\ 2\ ;\ 1+\log_3 7\}$

80. $5^x=\frac{391}{40-5^x}$ $\{R.\ \log_5 17\ ;\ \log_5 23\}$

75. $3^{4x}+2\cdot 9^x=143$ $\{R.\ 2\log_3 11\}$

82. $4^{3x}-8\cdot 4^{2x}+10\cdot 4^x+4=0$ $\{R.\ 1\ ;\ \frac{1}{2}\log_2\left(3+\sqrt{11}\right)\}$

83. $\frac{2^{-x}(2^{3x}-1)}{2^x-1}=3$ $\{R.\ assurda\}$

84. $2^{\sqrt{x+3}}\cdot 2^{\sqrt{31-x}}=256$ $\{R.\ 6\ ;\ 22\}$

DISEQUAZIONI ESPONENZIALI

85. $2^x>128$ $\{R.\ x>7\}$

86. $5^{x+2}<125$ $\{R.\ x<1\}$

87. $\left(\frac{2}{5}\right)^x>\frac{8}{125}$ $\{R.\ x<3\}$

88. $3^x<40$ $\{R.\ x<\log_3 40\}$

89. $9^x>243$ $\{R.\ x>\frac{5}{2}\}$

90. $2^{x+1}+2^x>48$ $\{R.\ x>4\}$

91. $3^{2x+3}+3^{2x+1}+3^{2x-1}>455$ $\{R.\ x>\frac{1+\log_3 5}{2}\}$

92. $\left(\frac{1}{2}\right)^{-x}<16$ $\{R.\ x<4\}$

93. $\left(\frac{4}{5}\right)^{x+1}+4\left(\frac{4}{5}\right)^{x+2}+5\left(\frac{4}{5}\right)^{x+3}>74$ $\{R.\ x<\log_{\frac{4}{5}}\frac{25}{2}\}$

94. $5^{2x}-130\cdot 5^x+625<0$ $\{R.\ 1<x<3\}$

95. $2\cdot 4^{2x+1}-513\cdot 4^x+64\geq 0$ $\{R.\ x\leq -\frac{3}{2}\vee x\geq 3\}$

96. $7^{2x}-54\cdot 7^x+245<0$ $\{R.\ \log_7 5<x<2\}$

97. $2^{6x}-7\cdot 8^{x+\frac{1}{3}}\geq 32$ $\{R.\ x\geq \frac{4}{3}\}$

98. $2^{2x+1} + 24\cdot 2^{-x} + 11\cdot 2^{x} > 37$ $\{R.\ x < 0 \vee x > \log_2 3 - 1\}$

99. $10\cdot 3^{4x} + 3^{3x} + 23\cdot 3^{2x} - 25\cdot 3^{x} + 6 < 0$ $\{R.\ \log_3 2 - \log_3 5 < x < -\log_3 2\}$

100. $\dfrac{b^{2x} - 2b^{-2x}}{5b^{2x} + b^{-2x}} > \dfrac{1}{10}$ $\{R.\ x > \frac{1}{4}\log_b \frac{21}{5}$ *se* $b > 1,\ x < \frac{1}{4}\log_b \frac{21}{5}$ *se* $0 < b < 1$, *assurda se* $b = 1\}$

EQUAZIONI LOGARITMICHE

Il simbolo "Log" indica il logaritmo in base 10

101. $\log_2 x = 7$ $\{R.\ 128\}$

102. $\log_3 (2x + 17) = 5$ $\{R.\ 113\}$

103. $\log_6 x + \log_6 (x - 1) = 1$ $\{R.\ 3\}$

104. $\log_2 (x + 3) + \log_2 (x^2 + 11x + 26) = 3$ $\{R.\ -7\ ;\ -5\ ;\ -2\}$

105. $\mathrm{Log}(x-1) + \mathrm{Log}(x+3) = \mathrm{Log}(2x+1)$ $\{R.\ 2\}$

106. $2\mathrm{Log}(x-1) - \mathrm{Log}(2x^2 + 3x - 5) = 0$ $\{R.$ *assurda*$\}$

107. $\mathrm{Log}(1 - x) - \mathrm{Log}(x + 5) = \mathrm{Log}(2x + 13) - \mathrm{Log}(2x + 9)$ $\{R.\ -4\ ,\ -\frac{7}{2}\}$

108. $\log_2 x + \log_2 (x + 2) - \log_2 (7 - x) = 1 + 2\log_2 3$ $\{R.\ 1\ ;\ 2 + \sqrt{22}\}$

DISEQUAZIONI LOGARITMICHE

109. $\log_5 x > -4$ $\{R.\ x > \frac{1}{625}\}$

110. $\log_6 x < 3$ $\{R.\ 0 < x < 216\}$

111. $\log_{\frac{1}{3}} (x + 4) \geq 2$ $\{R.\ -4 < x \leq -\frac{35}{4}\}$

112. $\log_3 x + \log_3 (9 - 2x) < 3$ $\{R.\ 0 < x < \frac{9}{2}\}$

113. $\log_2 (2x+1) + \log_2 \dfrac{5x+1}{7} > 4$ $\{R.\ x > 3\}$

114. $\log_7 (5x+1) - \log_7 \dfrac{11}{x+5} \leq 1$ $\{R.\ \frac{1}{5} < x \leq 2\}$

115. $\log_{\frac{9}{10}} (x-1) + \log_{\frac{9}{10}} (x-2) + \log_{\frac{9}{10}} (x-3) \leq \log_{\frac{9}{10}} (x^3 - 58)$ $\{R.\ \sqrt[3]{58} < x < 4\}$

116. $\log_3 (x^2 + 1) - \log_3 (x^2 - 1) > \log_3 13 - \log_3 12$ $\{R.\ -5 < x < -1 \vee 1 < x < 5\}$